Should the Strange Magnetic Moment of the Nucleon be Positive?

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ABSTRACT

The strange magnetic moment of the nucleon (μ_s) is examined as part of the nucleon's isoscalar anomalous moment. The dominant up and down quark effects in the anomalous moment may actually tend to favor $\mu_s > 0$, which is contrary to the negative values that generally result from model calculations. The possible origins of this apparent discrepancy are considered.

Several years ago, Kaplan and Manohar[1] pointed out that the neutral weak current could be used to determine the strange quark-antiquark ($\bar{s}s$) contributions to nucleon form factors. These $\bar{s}s$ effects are very interesting in that they represent low energy properties of the nucleon which must be due to the quark-antiquark sea. Determining such quantities can thus provide deeper insight into the origins of nucleon properties in terms of QCD.

The magnetic moment is an excellent example of a nucleon property that can be studied in this fashion. Electromagnetic couplings probe only the charged constituents of the nucleon: the quarks and antiquarks. The anomalous components of the nucleon magnetic moments are known to be large, but we still do not have a satisfactory explanation of their magnitudes in the context of QCD. While many models do give reasonable values, none are firmly based (without additional assumptions) on QCD. Thus, the real origin of the nucleon's anomalous magnetic moment in terms of QCD (rather than ad hoc models like constituent quarks) has been elusive.

After the suggestion of Kaplan and Manohar, it was proposed that the neutral weak magnetic moment of the proton could be measured in parity violating electron scattering.[2] Indeed, such an experiment (the SAMPLE experiment[3]) is currently in progress and results are expected soon.

In this Letter I examine the role of μ_s, the strange quark-antiquark contribution to the nucleon's isoscalar anomalous moment. In particular, it will be suggested that it may be very natural to expect that the value of μ_s is positive, rather than the generally negative values obtained in model calculations.[4−9]

In general, we can define the vector matrix elements corresponding to each quark flavor as follows[10]:

$$< N|\bar{q}_f\gamma^\mu q_f|N > \equiv \bar{u}_N \left[F_1^f(k^2)\gamma^\mu + \frac{i}{2M_N}F_2^f(k^2)\sigma^{\mu\nu}k_\nu \right] u_N \tag{1}$$

in which q_f is a quark field operator of flavor $f = u, d, s$, and the F_1^f and F_2^f are form factors which are functions of the squared momentum transfer. The electromagnetic nucleon form factors are then obtained as functions of the individual flavor form factors:

$$F_{1,2} = \frac{2}{3}F_{1,2}^u - \frac{1}{3}F_{1,2}^d - \frac{1}{3}F_{1,2}^s \tag{2}$$

where the coefficients represent the electromagnetic couplings to the individual quarks (i.e., the electric charges). Similarly, the neutral weak couplings can be written

$$F_{1,2}^Z = (\frac{1}{4} - \frac{2}{3}\sin^2\theta_W)F_{1,2}^u + (-\frac{1}{4} + \frac{1}{3}\sin^2\theta_W)F_{1,2}^d + (-\frac{1}{4} + \frac{1}{3}\sin^2\theta_W)F_{1,2}^s . \tag{3}$$

(Radiative corrections slightly modify this tree level expression.[11]) Thus we can see that a determination of the isoscalar, isovector, and neutral weak form factors would allow solving for the contributions of all 3 flavors.

In the following, we will explore the use of eq. (2) at $k^2 = 0$ and the implications of the well-known electromagnetic moment values for the flavor components of the anomalous moments. Beginning first with

F_1 (equal to the nucleon electric charge at $k^2 = 0$), we find

$$Q_p = \frac{2}{3}F_1^u - \frac{1}{3}F_1^d$$
$$Q_n = \frac{2}{3}F_1^d - \frac{1}{3}F_1^u \tag{4}$$

where we have assumed isospin symmetry and define $F_1^u \equiv F_1^u(\text{proton}) = F_1^d(\text{neutron})$, $F_1^d \equiv F_1^d(\text{proton}) = F_1^u(\text{neutron})$. Note also that we have set $F_1^s = 0$ since there is no net contribution of strange quarks to the nucleon charge. (This is *not* true for the magnetic moment.)

Using eq. (4) we obtain the trivial results $F_1^u = 2$ and $F_1^d = 1$, which of course are just the valence numbers of the quarks. Now let's apply eq. (2) to the anomalous moments. Then we obtain

$$F_2^p = \frac{2}{3}F_2^u - \frac{1}{3}F_2^d - \frac{1}{3}\mu_s$$
$$F_2^n = \frac{2}{3}F_2^d - \frac{1}{3}F_2^u - \frac{1}{3}\mu_s \ . \tag{5}$$

Using the empirical values of $F_2^p = 1.79$ and $F_2^n = -1.91$, and forming isovector and isoscalar combinations, we get

$$3.70 = F_2^u - F_2^d$$
$$-0.12 = \frac{1}{3}(F_2^u + F_2^d) - \frac{2}{3}\mu_s \ . \tag{6}$$

We have 2 equations and 3 unknowns. Determination of μ_s would then allow solving for the other 2 flavor form factors.

Nevertheless, we can gain some insight from eq. (6). First, one should note that the left side of the isoscalar equation is small compared to the isovector. Since we do not expect μ_s to be large (generally $|\mu_s| < 1$), we see that roughly $F_2^u \approx -F_2^d \sim 2$. It is striking that these anomalous moments associated with the up and down quark flavors are so large but similar in absolute magnitude. From eq. (6) we see that the relative magnitudes of F_2^u and F_2^d are closely related to the sign of μ_s So the next question we might ask is whether we expect $|F_2^u| > |F_2^d|$ or vice versa.

Our knowledge of the flavor dependence of other nucleon properties indicates that the up quark effects are generally dominant. Valence quantities clearly show this effect due to the fact that there are 2 valence up quarks and only one valence down quark. For example, this is manifestly true in the valence quark counting involved with $F_1^u = 2F_1^d$. Another indication comes from the axial matrix elements,

$$\Delta q_f = - <N|\bar{q}_f\gamma^\mu\gamma^5 q_f|N> S_\mu \tag{7}$$

where S_μ is the nucleon spin vector. These matrix elements are probably more relevant to the F_2^f since they also involve the spin couplings of the quarks. Recently, spin dependent deep inelastic scattering experiments[12] have determined $\Delta u = 0.85 \pm 0.03$ and $\Delta d = -0.41 \pm 0.03$, so the spins of the up quarks are dominant. If F_2^u and F_2^d have similar behavior, then we would have $|F_2^u| > |F_2^d|$.

One area where it has been found that the up quarks do not dominate is in the antiquark sea. Measurements of the Gottfried sum rule indicate that[13]

$$\int_0^1 dx[\bar{u}(x) - \bar{d}(x)] = -0.147 \pm 0.039 \ . \tag{8}$$

Here $\bar{u}(x)$ and $\bar{d}(x)$ are deep inelastic structure functions corresponding to the antiquarks, and are functions of the momentum fraction x. However, this is a pure sea quantity with no valence contribution at all. In addition, there is no spin dependence in these quantities. It would seem that the dominance of the up quarks in the axial matrix elements is more relevant to the magnetic moment discussion.

Regarding the magnetic moments, there is no apriori requirement that the compositeness of the nucleon should generate large anomalous magnetic moments (i.e., $F_2^u = F_2^d = 0$ would certainly be allowed in

principle). Clearly, the quantities F_2^u and F_2^d are generated dynamically and this is a very large (not subtle) effect. Thus it would seem that we should expect these quantities to display the dominance of up quarks that we discussed above, leading to $|F_2^u| > |F_2^d|$.

Now we return to eq. (6). If $|F_2^u| > |F_2^d|$ holds, then the isoscalar equation implies that $\mu_s > 0$. In fact, if $\mu_s < 0$ then one is forced to conclude that $|F_2^d| > |F_2^u|$. So we see that it actually seems more natural to expect $\mu_s > 0$.

Table 1 shows the values of F_2^u and F_2^d for some typical values of μ_s. Note that a negative value of μ_s causes a reversal in the relative magnitudes of F_2^u and F_2^d, which is contrary to the expectations based on the above discussion. Therefore, it seems that a positive value of μ_s is a rather natural expectation based on these considerations.

Table 1

μ_s	F_2^u	F_2^d
$-\frac{1}{2}$	1.17	-2.53
0	1.67	-2.03
$\frac{1}{2}$	2.17	-1.53

In general, model calculations[4-9] of μ_s give $\mu_s < 0$. (I do note that one reference[14] does predict a positive $\mu_s = 0.42 \pm 0.3$ based on an SU(3) chiral hyperbag model.) These models typically invoke a specific mechanism, such as the fluctuation of a proton into a Λ and K^+, for generating μ_s. However, such dynamical mechanisms are probably only part of the total strange magnetic moment. For example, the constituent quarks in the baryons may have intrinsic $\bar{s}s$ pairs which are not dynamical degrees of freedom in baryon-meson or constituent quark models. Perhaps there is a deeper reason why the constituent quarks themselves may make a positive contribution to μ_s. Clearly, the experimental determination of this quantity will be extremely interesting and will provide an important new clue to the structure of the nucleon.

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